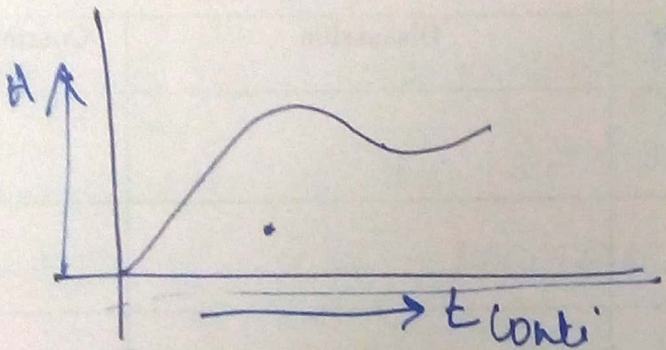
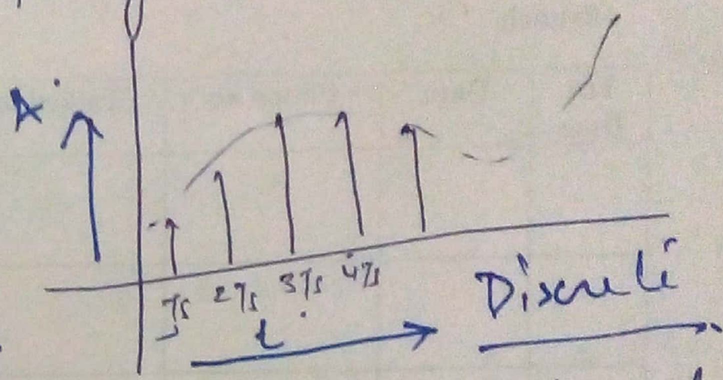
# Unit - V →

**Sampling**: It is a process of converting a continuous-time signal $x(t)$ into a discrete time signal $x(n)$ by measuring the amplitude of continuous time signal $x(t)$ at integer multiples of a sampling interval $T(s)$



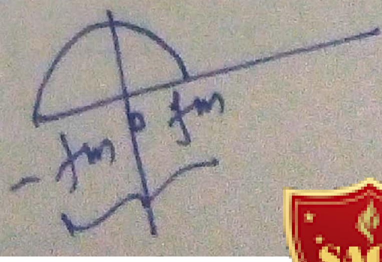

**Sampling Theorem**: It states that a band limited signal of finite energy, which has no frequency components higher than $2f_m$ may be recovered from its sampling freq

$f_m$ $\quad BW = 2f_m$

$f_s \geq 2f_m$

The minimum sampling rate or min. sampling freq. $\boxed{\omega_m = 2f_m}$ - Nyquist rate

$$\left[ T_s = \frac{1}{2f_m} \right]$$




$f_s > 2f_m$.

From Sampling Theorem

(1) A band-limited signal of finite energy which has no freq.- component higher than $f_m$ Hz is completely described by its sample value at uniform intervals less than or equal to $\frac{1}{2f_m}$ sec apart

(2) May be completely recovered from the knowledge of its samples taken at the rate of $2f_m$ samples per second.

⇒ (1) continuous time signal — samples

(2) Reconstruction of original signal from its sample.

Proof: Let us consider a continuous time signal whose spectrum is band limited to $f_m$ Hz. ∴ $X(\omega) = 0$ for $|\omega| > \omega_m$

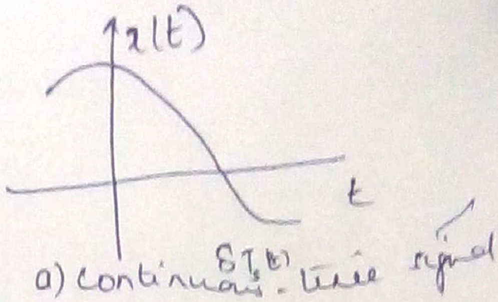
Let $x(t)$ continuous time signal

sampling of $x(t)$ at a rate of $f_s$ Hz. (samples per second) may be achieved by multiplying $x(t)$ by an impulse train $\delta_{Ts}(t)$

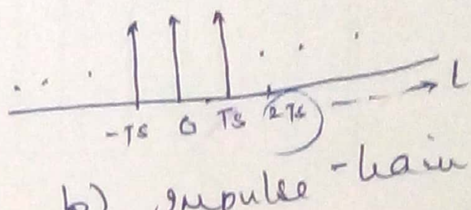
It consists of unit impulses repeating periodically ...

The resulting or sampled signal

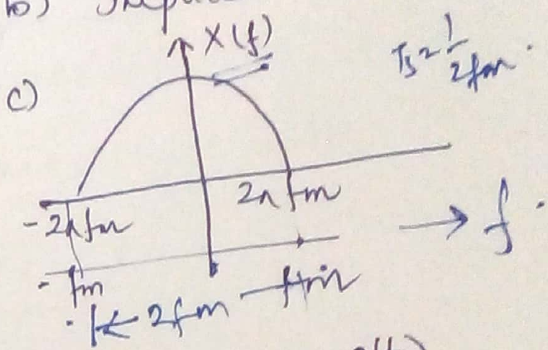
$$g(t) = x(t)\,\delta_{Ts}(t) \quad (1)$$



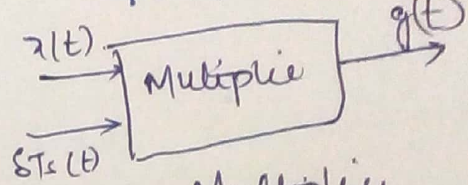

As impulse train is a periodic signal of period Ts so it can be expressed in terms of Fourier series



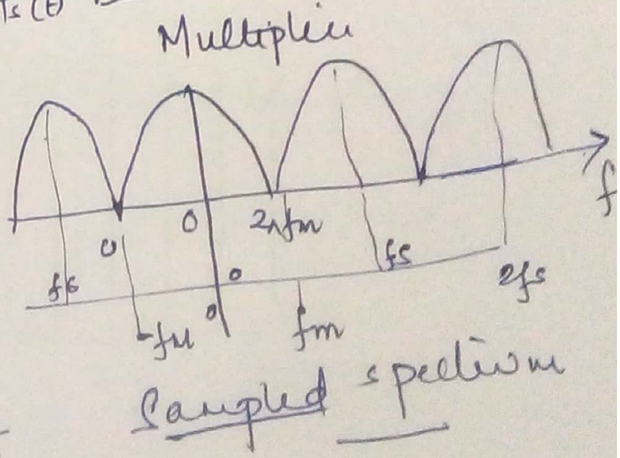

$$\delta_{Ts} = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 3\cos\omega_s t + \cdots\right] \quad (2)$$




$$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$




Putting value of $\delta_{Ts}$ from eqn (2) in eqn (1)




$$g(t) = \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 3x(t)\cos\omega_s t + \cdots\right] \quad (3)$$

Now to obtain freq spectrum take Fourier transform of eqn (3)

$$G(\omega) = \frac{1}{T_s}\Big[X(\omega) + [X(\omega-\omega_s) + X(\omega+\omega_s)] + X(\omega-2\omega_s) + X(\omega+2\omega_s) + \ldots$$

$\delta(t) \rightleftharpoons 1$

$\cos\omega_s t \rightleftharpoons \frac{1}{2}[e^{\ldots}$

$e^{j2\pi f_c t} \rightleftharpoons$

$\mathcal{F}[x(t)\cos 2\pi f_s t] \rightleftharpoons \left[\frac{1}{2}X(f-f_s) + X(f+\ldots\right.$

$$\boxed{G(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)}$$

$G(t)$

Spectrum $G(\omega)$ consists of $X(\omega)$ repeating periodically with period

$\omega_s = \frac{2\pi}{T_s}$ rad/sec,

or $f_s = \frac{1}{T_s}$ Hz.

Nyquist (+)

$\boxed{f_s > 2f_M}$ no overlapping occurs

$f_s = 2f_M$ — but touching

$f_s < 2f_M$ — overlapping

$$x(t)\cdot\cos\omega_c t \rightleftharpoons \frac{1}{2}\left[e^{j\omega_c t} + e^{-j\omega_c t}\right]$$

$j\omega_c t$

$$x(t)\, e^{\cdot} \rightleftharpoons X(\omega - \omega_c)$$

freq. shift.